

Market Freeze and Recovery: Trading Dynamics under Optimal Intervention by a Market-Maker-of-Last-Resort*

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Abstract

We study the trading dynamics in a distressed asset market with search frictions. When trading of a financial asset ceases due to an adverse selection problem, a large player can resurrect the market by buying up bad assets which involves assuming financial losses. The player can, however, delay the intervention: a mere announcement today of intervening at a later point in time can cause markets to function again. This announcement effect gives rise to a trade-off between the size and the timing of the intervention. The optimal intervention involves balancing the financial losses from the intervention and the social cost of illiquid markets. If the losses are small and a market is deemed important, it is optimal to ensure that the market functions continuously. In this case, there is a fixed cost associated with intervention delay, making it optimal to intervene as early as possible at the minimum size. As losses increase and the importance of the market declines, the intervention is optimally delayed and it can be optimal to rely on the announcement effect by increasing the size of the intervention.

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Furthermore, our finding highlights the importance of search friction in the formation of market distress, the determination of policy announcement effect, and the optimal design of intervention.

1 Introduction

The financial system has traditionally relied on direct lending between individual institutions and investors for financing and allocating liquidity. More recently, however, liquidity provision has moved away from intermediaries to markets. Examples are primary and secondary markets for mortgage-, other asset-backed securities and for commercial paper or the interbank market. Liquidity problems do not arise because an individual institution cannot find funding. Instead, they manifest themselves as entire markets for assets shutting down – or as a “market freeze” – with financial institutions being unable to access them for liquidity.

The financial crisis of 2007-2009 forced governments and central banks to heavily intervene in financial markets that were impaired by the risk and uncertainty surrounding the assets being traded. A prominent intervention was the outright purchase of assets in distressed markets either permanently or through term repurchase operations.¹ These measures were unprecedented and policy makers were forced to act without much guidance from economic research for their appropriateness and design. As liquidity provision through markets will remain prominent, it is necessary to gain a better understanding of how to engage in such *market-making-of-last-resort* and to develop guidelines for when such policies are indeed optimal.

In this paper, we study how a market where trading has ceased reacts to the intervention of a large player and analyze the optimal design of the intervention when the liquidity crisis is dynamically unfolding.² To generate a market freeze endogenously, we introduce adverse selection with respect to the quality of the assets traded into a search-theoretic model of asset pricing. More specifically, investors search for trading partners to allocate liquidity. Investors sell assets if they need liquidity, while other investors are willing to provide such

¹The Fed for example is currently purchasing mortgage-backed securities and offers other programs that are technically lending facilities, but in fact remove toxic assets from markets for a horizon of up to 5 years. All these measures placed assets from markets where trading had ceased onto the asset side of the Fed’s balance sheet (see Bernanke [1]).

²In principle, any public entity could assume such a function. As the current crisis, however, shows, central banks and possibly Treasury Departments are the most likely institutions to carry out such policies.

liquidity by buying the asset. The quality of the asset varies, and investors who own the asset can observe its quality, but potential buyers cannot. This causes an adverse selection problem for trading the asset: Bad assets are always on the market, while good assets are only on the market if an investor needs to sell to obtain liquidity. When the average quality of the asset falls unexpectedly and permanently, the adverse selection problem becomes more severe causing trading of the asset to stop – the market freezes.

With the market not functioning, investors cannot obtain liquidity anymore by selling the asset. Over time the costs of the market freeze increase, as more and more investors need to raise liquidity but cannot access the market. A large player called *market-maker-of-last-resort* (MMLR), however, can intervene at some time after the shock by buying bad assets. We show that the MMLR can establish trading after the intervention again, if he removes a sufficiently large quantity of bad assets which involves assuming losses: he needs to offer a price to lemons (investors that hold a bad asset) that makes them at least indifferent between selling to the MMLR at the time of the intervention and waiting to sell to other investors when the market functions again.

When characterizing the dynamics of trading before and after the intervention, we uncover an *announcement effect*: when the MMLR announces a future intervention, investors might have an incentive to trade before the intervention. This implies that the market can recover fully or partially before the actual intervention. The reason is quite intuitive. First, the intervention offers a (discounted) option value when buying the asset. Having bought a lemon, an investor will have a chance to sell it to the MMLR in the future. Second, there is a strategic complementarity. The intervention resurrects the market, implying that an investor can sell an asset on the market after the intervention. Finally, if trading had stopped for some time, the average quality of the asset has increased, as more and more investors are trying to sell a good asset, but were not able to do so.

We characterize the time when the market recovers as a function of the policy variables which are the time of intervention and the option value. The MMLR can raise the option value by increasing the purchase price and the purchased amount of bad assets. The option value needs to be financed by transfers which correspond to the costs of the intervention. An optimal policy balances the net present value of losses from purchasing bad assets against the benefits for investors from being able to obtain liquidity again through asset sales. Interestingly, the announcement effect causes a trade-off between the size of the intervention and its timing: announcing a larger intervention is more costly, but allows markets to recover faster.

If a MMLR is required to maintain continuously functioning markets, we show that the

optimal policy is to intervene immediately after the shock, but at the minimum scale that is required to resurrect the market. We then study the optimal size and time for intervening which involves balancing the financial losses from the intervention and the social cost of illiquid markets. We find that, if the losses are small and a market is deemed important, it is optimal to ensure that the market functions continuously by an immediate intervention. This is in stark contrast to the literature that analyzes optimal marketmaking in the face of pure liquidity shocks.³ As losses increase and the importance of the market declines, the intervention is optimally delayed and it can be optimal to rely on the announcement effect by increasing the size of the intervention.

Employing a framework based on search in asset markets⁴ is not only a matter of convenience, but captures several features that are crucial for analyzing market-making-of-last-resort. First, such a framework has a clear notion of liquidity and its cost, which corresponds to how easy it is to sell an asset in the market.⁵ Second, the framework is dynamic allowing us to study how policy and trading on the market interact with each other. Third, allocating liquidity involves a (dynamic or intertemporal) strategic complementarity often described as liquidity hoarding. This is also present in our framework, as the ability to sell the asset in the future influences the incentives to buy an asset (in other words, provide liquidity) today. Our finding highlights the importance of search friction in the formation of market distress, the determination of policy announcement effect and the design of optimal intervention.

There is an emerging literature that studies the effects of liquidity crises in search models of asset pricing. Weill [18] exhibits the role dealers play in over-the-counter markets to absorb temporary selling pressures. Even more relevant is the work by Lagos et al. [12]. Their paper analyzes when private dealers do not have sufficient incentives to provide liquidity to investors in times of a crisis. While pointing out that a government may improve upon the situation with a small intervention, they do not study the strategic interaction of the government as a large player with the market. Furthermore, the authors abstract from the fundamental reasons for the crisis. These are important, however, as they determine the losses the MMLR will have to absorb as a result of its intervention.

In our approach, a liquidity crisis arises due to an unforeseen shock to the average quality of assets being traded, which causes the market for these assets to freeze as a result of an

³For example, Weill [18] shows that in the case of a liquidity shock, it is never optimal to intervene immediately.

⁴See for example Duffie et al. [6], [7], Vayanos and Wang [16] or Vayanos and Weill [17].

⁵Such models are also a good description of many markets where financial institutions obtain short-term liquidity such as overnight markets, money markets and markets for securitized assets.

adverse selection problem.⁶ Recently, there have been other, fundamental explanations for such a crisis. Brunnermeier and Pedersen [4] point out that margin requirements can lead to liquidity shortages in markets. Bolton et al. [2] identify strategic considerations where investors obtain liquidity through asset sales early in the anticipation of a potential crisis, thus precipitating the crisis itself by selling assets at fire sale prices. While the first paper builds on a coordination problem, the latter has investors trade early to prevent being caught in an adverse selection problem.⁷

It is worth to briefly discuss our notion of a MMLR. Marketmakers provide liquidity by standing ready to accumulate or draw from an inventory of assets to smooth out fluctuations in the demand and supply for the assets. Usually working for profit, they manage their asset portfolio by charging a bid-ask spread on their transactions with market participants.⁸ In a crisis situation, where due to uncertainty or risk a market freezes, private marketmakers that operate strictly for profit might neither have the capital, nor the incentives to provide such a function. An institution like a central bank, however, could act as a marketmaker and stabilize markets even if this involves expected losses.

The notion of a MMLR as a public entity that intervenes in some market is relatively new (see for example Buiter [5]), and a detailed discussion of how to perform such a function is largely absent in the literature. Such a function is different from regular marketmaking. First, a MMLR is only transacting in extreme situations such as a crisis where the entire market breaks down for a long time. In normal times, the MMLR is not involved in trading, as there are no gains from intervening. Second, a MMLR will in general not be able to run down an inventory of assets that he has accumulated during a crisis. This implies that he will not necessarily be able to operate for profit. Third, he is non-competitive acting as a large player at a specific time in the market. Summarizing, a MMLR acts essentially as the *buyer-of-last-resort* with the aim of resurrecting the market. Still, he does provide liquidity in the sense that he stands ready to absorb bad assets in times of a crisis. We recover all these features in our model.⁹

⁶As such, our paper also contributes to the growing literature on search and the role of private information (see for example Guerrerri et al. [8], Lester et al. [13], Rocheteau [14]) or Williamson and Wright [19].

⁷Another paper by Heider et al. [9] stresses adverse selection concerning counterparty risk as an explanation for a breakdown in interbank lending through liquidity hoarding. Their static model can not, however, study explicitly the interaction between policy interventions and market recovery.

⁸See for example Rust and Hall [15] or Duffie et al.[6].

⁹A MMLR is different from a private investor in different aspects. A MMLR is able to commit to future actions, is able to overcome search frictions, is sufficiently large to affect market outcome, and is benevolent (and thus internalize trading externalities).

The role of a government providing liquidity for the entire financial system has not been widely studied in the literature. Holmström and Tirole [10] is an early example where the state-contingent supply of public debt such as Treasuries can help to insure against aggregate uncertainty and can be interpreted as public liquidity provision in a crisis. More closely related to the proposed research, Bolton et al. [3] demonstrate that a government that makes a market for assets can support a price floor for the asset. Such a policy removes the incentives for asset sales at early stages of a potential liquidity crisis, thereby ensuring well-functioning markets. This has also implications for the timing of public liquidity: to prevent fire sales of assets, the MMLR must be committed to provide the floor in the future when liquidity might dry up due to an adverse selection problem. We focus here on the design of the intervention taking into account how the market will react to it before, during and after the intervention.

The paper proceeds as follows. Section 2 introduces our model. In Section 3 we analyze how an unanticipated permanent negative shock to the asset quality can lead to a market freeze. Section 4 studies how a MMLR can intervene to maintain a continuously functioning market. Section 5 derives the optimal time and size of intervention in different situations. In Section 6 we extend the model to discuss the optimal intervention when the quality shock is temporary and random.

2 Economic Environment

2.1 Overview

We employ a simple model of asset pricing under search frictions. The initial step is to introduce one complication into such a model. There is an adverse selection problem, as assets differ in their payoff structure which is private information for the owner of the asset. As described below, this model will be capable of generating a partial or full breakdown of trading through a combination of liquidity needs and information problems. We then add a large player (called a MMLR) to the model that is confronted with such a market breakdown. This large player can set a price and choose a time to buy any specific quantity of assets on the market. This will enable us to study how one optimally trades off the benefits from helping the market to recover with the potential losses from doing so.

2.2 Model

Time is continuous and there are S assets. These assets are of two types. A fraction π of the assets yields a dividend δ , whereas the rest does not yield a dividend. The return on these assets is private information for the owner of the asset; i.e., only the holder of the asset can observe its return, but not other investors. The assets are traded on the same market as they are indistinguishable for a potential buyer, even though the seller knows whether they yield a return or not.

Investors are risk-neutral and discount time at a rate r . We assume that each investor can either hold one unit of an asset or no assets. They differ according to their valuation of the asset: if the asset yields a dividend, they either have a holding benefit of $x_b \geq 0$ (and, hence, are potential buyers of the asset) or a holding cost of $x_s \in (0, \delta)$ (and, hence, are potential sellers of the asset).¹⁰ Investors switch their valuation of the security from x_b to x_s according to a Poisson process with arrival rate $\kappa \in \mathbb{R}_+$. This captures the fact that some investors who own a security might have a need for selling it – or in other words, have a need for liquidity. The higher κ , the more likely an investor will face such needs.

To facilitate the analysis, we assume here that investors that sell the security leave the economy and are replaced by new potential buyers. Similarly, when a potential buyer does not own a security, but switches in its valuation of a security to a holding cost, he will leave the economy being replaced by a new investor.

We assume throughout that the number of buyers, μ_b , is given exogenously and is not influenced by the intervention of the MMLR. This is akin to assuming that there is a large number of potential buyers relative to the assets in the market. Hence, we assume that the number of buyers stays constant over time ($\dot{\mu}_b = 0$).¹¹ We assume further that there is no centralized market mechanism to trade assets, but that investors are matched according to a technology given by a matching function $M = \lambda\mu_b[\mu_s + \mu_\ell]$, where M is the total number of matches, λ is a parameter capturing the matching rate, and (μ_s, μ_ℓ) are the endogenously arising fractions of sellers with assets yielding a dividend and not yielding a dividend, respectively.¹² Our last assumption is that in pairwise meetings, only the buyer

¹⁰ While the holding cost proxies for liquidity needs, additional benefits of the asset can arise for example from its value as collateral backing other investments.

¹¹ This enables us to keep the law of motion a system of *linear* differential equations.

¹² The interpretation is that investors are matched according to a Poisson process with a fixed arrival rate. As a result, matches with investors seeking the opposite side of the trade occur at a rate λ which is proportional to the measure of that investor group.

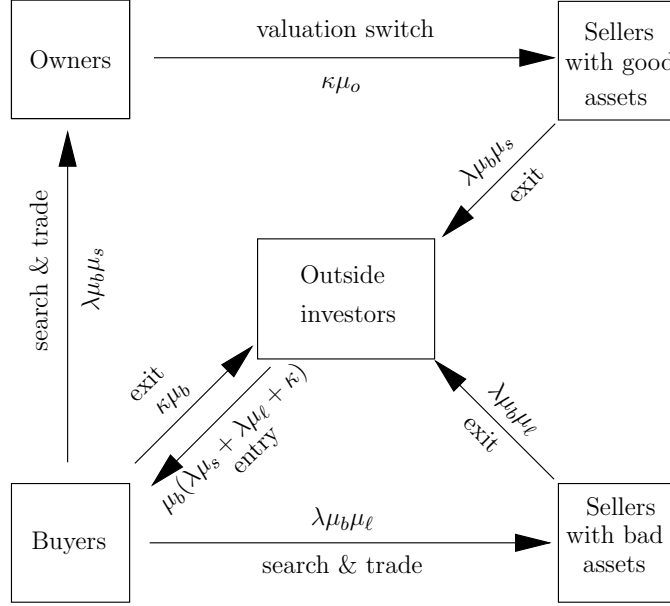


Figure 1: Flow Diagram

can make a take-it-or-leave it offer.¹³

We can then describe the economy by the flow diagram in Figure 1. There are four groups of investors: buyers (b), inactive owners (o), investors with good assets that want to sell (or sellers) (s) and investors with bad assets (or “lemons”) (ℓ). Investors enter the economy from the pool of outside investors, and investors exiting the economy return to that pool. A buyer becomes an owner by buying a good asset (with probability $\lambda\mu_s$), becomes a lemon by buying a bad asset (with probability $\lambda\mu_\ell$) and exits the economy by switching the valuation (with probability κ). An owner also becomes a seller when switching his valuation (with probability κ). Finally, if there is trade, sellers and lemons sell their assets and exit the economy (with probability $\lambda\mu_b$). Note that lemons choose to transit immediately from buying to selling the asset, while owners have first to experience a liquidity shocks. This gives rise to a classic adverse selection problem.

¹³ This is a simplifying assumption to avoid the issue of bargaining in the presence of imperfect information. See for example [13].

3 Market Freeze

3.1 Pricing and Value Functions

To trade assets, agents can produce a numeraire good with which they have a linear utility of consumption and production.¹⁴ In any meeting, a buyer can make a take-it-or-leave-it offer to buy the security at price p . As the buyer cannot observe the type of security traded, he will have to infer from his offer whether sellers with good assets will sell the security or not. Sellers will demand at least a price that corresponds to the value of holding on to the security. For sellers with bad assets, this is the expected resale value of the asset. For sellers with good assets, this is the expected resale value plus the holding benefits. The problem of adverse selection together with risk neutrality implies then that all sellers will sell at the same price; in other words, they pool.¹⁵

We first derive the value functions for sellers with a good security, denoted by $v_s(t)$ and determine then the take-it-or-leave-it offer of the buyer. Denoting the first time a seller meets a buyer by τ , we obtain

$$v_s(t) = E_t \left[\int_t^\tau e^{-r(s-t)} (\delta - x_s) ds + e^{-r(\tau-t)} \max\{p(\tau), v_s(\tau)\} \right]. \quad (3.1)$$

The first expression on the right-hand side is the flow value from owning the security. The second term gives the discounted value of meeting a buyer at random time $\tau > t$. In such a meeting, the seller either accepts the offer at price p or rejects it, staying as a seller (receiving value $v_s(\tau)$). Differentiating this expression with respect to time t and rearranging yields

¹⁴Alternatively, we can assume that agents can transfer utility. This assumption implies that buyers can finance all profitable purchases.

¹⁵ One can show that there is no separating equilibrium even using lotteries. The intuition is that separating offers cannot be an equilibrium outcome because (i) buying only bad assets is not profitable because trade surplus is zero; (ii) buying only good assets is not feasible because sellers have a tighter participation constraint than lemons; (iii) buying both assets using separating contracts requires offering a higher price and a higher trading probability to lemons, which turns out to be dominated by a pooling offer.

the following differential equation¹⁶

$$rv_s(t) = (\delta - x_s) + \gamma(t)\lambda\mu_b \max\{p(t) - v_s(t), 0\} + \dot{v}_s(t), \quad (3.2)$$

where $\gamma(t) \in [0, 1]$ is the probability that the buyer makes a take-it-or-leave-it offer to the seller in order to buy the security. Hence, we allow buyers to randomize between making a take-it-or-leave-it offer (probability γ) and not trading (probability $1 - \gamma$) when meeting a seller.

As we will see below, the price of asset $p(t)$ is constant over time, and thus upon receiving a bad asset, a buyer will immediately want to sell the asset. We denote the value function of a lemon by v_ℓ . Similarly, upon receiving a good asset, a buyer will never sell it in a pooling equilibrium, as he has a holding benefit x_b and as any new potential buyer faces the risk of acquiring a lemon. We denote the value functions of owners by v_o . The value functions are given by

$$rv_\ell(t) = \gamma(t)\lambda\mu_b \max\{p(t) - v_\ell(t), 0\} + \dot{v}_\ell(t) \quad (3.3)$$

$$rv_o(t) = (\delta + x_b) + \kappa(v_s(t) - v_o(t)) + \dot{v}_o(t) \quad (3.4)$$

$$rv_b(t) = \lambda(\mu_s(t) + \mu_\ell(t)) \max\{\max_p \tilde{\pi}(p)v_o + (1 - \tilde{\pi}(p))v_\ell(t) - p - v_b(t), 0\} \quad (3.5)$$

$$-\kappa v_b(t) + \dot{v}_b(t), \quad (3.6)$$

where the last expression takes into account that the buyer in a meeting can opt not to buy the asset and that – provided he makes an offer – he will choose a price that maximizes his expected pay-off given the adverse selection problem. This implies that he takes into account the probability of having a good asset *conditional* on his offered price ($\tilde{\pi}(p)$).

An equilibrium is given by value functions and prices such that the take-it-or-leave-it-offer is optimal for buyers and accepting or rejecting the offer is optimal for sellers and lemons for all t . In a pooling equilibrium, all assets are traded at the same price. Sellers with good assets will only sell if they get at least their outside option of holding the asset forever. The buyers,

¹⁶To gain some intuition, it is useful to use an approximation. For small Δ , we have

$$v_s(t) = \frac{1}{1 + r\Delta} \left[(\delta - x_s)\Delta + \gamma(t + \Delta)\lambda\mu_b\Delta \max\{p(t + \Delta), v_s(t + \Delta)\} + (1 - \gamma(t + \Delta))\lambda\mu_b\Delta v_s(t + \Delta) + o(\Delta) \right].$$

The first term on the right-hand side is the flow utility in a small time interval Δ until the first buyer makes an offer. The second term expresses the fact that with probability $\lambda\mu_b\Delta$ the seller meets a potential buyer who makes an offer (with probability γ) with value $\max\{p(t + \Delta), v_s(t + \Delta)\}$ for the seller. This takes into account that the seller has the option to decline the offer. The final expression expresses the fact that the seller does not meet a potential buyer. Rearranging and letting $\Delta \rightarrow 0$, we again obtain the expression in the text.

however, will never offer more than this outside option. Hence, in equilibrium whenever an asset is traded

$$p = v_s = \frac{\delta - x_s}{r}. \quad (3.7)$$

This is the standard pricing from the literature on search and asset prices taking into account our assumptions on bargaining. Prices are equal to the discounted net present value of dividends minus a liquidity premium.

3.2 Steady State Equilibria

We now derive values for the average quality of the assets, π , for which all assets are traded and for which none are traded in a steady state equilibrium. In a steady state equilibrium, the buyers strategy γ will pin down a distribution of assets across investors according to the flow equations

$$\dot{\mu}_b = 0 \quad (3.8)$$

$$\dot{\mu}_s = \kappa\mu_o - \gamma\lambda\mu_b\mu_s \quad (3.9)$$

$$\dot{\mu}_\ell = -\gamma\lambda\mu_b\mu_\ell + \gamma\lambda\mu_b\mu_\ell = 0 \quad (3.10)$$

$$\dot{\mu}_o = -\kappa\mu_o + \gamma\lambda\mu_b\mu_s. \quad (3.11)$$

Note that buyers that acquire lemons will immediately try to sell the asset in a pooling equilibrium. This implies that the number of lemons that are being sold are constant. In steady state, we obtain

$$\mu_o + \mu_s + \mu_\ell = S \quad (3.12)$$

$$\mu_o + \mu_s = \pi S \quad (3.13)$$

$$\kappa\mu_o = \gamma\lambda\mu_b\mu_s. \quad (3.14)$$

where the first two equations are just accounting identities for all assets and good assets.

Given the pooling equilibrium, the probability of obtaining a good asset $\tilde{\pi}$, due to the adverse selection problem is given by

$$\tilde{\pi} = \begin{cases} \frac{\mu_s}{\mu_s + \mu_\ell} & \text{if } p \geq v_s(\delta) \\ 0 & \text{if } p < v_s(\delta). \end{cases} \quad (3.15)$$

where the steady state measures of asset holders are given by

$$\begin{aligned}\mu_s &= \frac{\kappa}{\gamma\lambda\mu_b + \kappa} S\pi \\ \mu_o &= \frac{\gamma\lambda\mu_b}{\gamma\lambda\mu_b + \kappa} S\pi \\ \mu_\ell &= (1 - \pi)S.\end{aligned}$$

Therefore, if there is trade of the good asset in steady state we have

$$\tilde{\pi} = \frac{\kappa\pi}{\kappa + (1 - \pi)\gamma\lambda\mu_b}. \quad (3.16)$$

Hence, equation (3.16) captures a negative “quality” effect. When more investors buy the asset (γ increases), there are fewer good assets available for sale, reducing a buyer’s incentive to buy at the price $p = (\delta - x_s)/r$. To facilitate notation, we normalize μ_b to 1 from now on.

We consider next the optimal strategy of the buyers. Buyers trade if and only if

$$\tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - v_s \geq v_b \quad (3.17)$$

where

$$v_b = \frac{\lambda(\mu_s + \mu_\ell)}{\lambda(\mu_s + \mu_\ell) + (r + \kappa)} [\tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - v_s]. \quad (3.18)$$

Hence, they trade if and only if

$$\tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - v_s \geq 0. \quad (3.19)$$

The value of holding assets will depend on whether an inactive owner is able to sell the asset again or not if he switches to become a seller – or, equivalently, on the strategy of the buyer, γ .

$$v_o = \frac{1}{r + \kappa} [(\delta + x_b) + \kappa v_s] \quad (3.20)$$

$$v_\ell = \frac{\gamma\lambda}{\gamma\lambda + r} v_s. \quad (3.21)$$

Note that the value of holding a good asset is independent of γ . The reason is that without selling it, one derives still a fundamental value from it. For bad assets, the value depends purely on the ability of selling it again. If this is the case, one realizes some surplus from

pooling with owners of the good assets. Rewriting condition (3.19) we get for trade

$$\tilde{\pi}(\delta + x_b) - (\delta - x_s) \left[1 + (1 - \tilde{\pi}) \frac{\kappa - \lambda\gamma}{r + \lambda\gamma} \right] \geq 0 \quad (3.22)$$

or

$$\tilde{\pi} \left[\frac{\delta + x_b}{\delta - x_s} r + \kappa + \lambda\gamma \left(\frac{\delta + x_b}{\delta - x_s} - 1 \right) \right] \geq r + \kappa. \quad (3.23)$$

First, set $\gamma = 1$ and define $(\delta + x_b)/(\delta - x_s) = \xi$. Then, to get trade we need

$$\pi \geq \frac{(\kappa + \lambda)(r + \kappa)}{\kappa(\xi r + \kappa) + \lambda(\xi\kappa + r)} \equiv \bar{\pi}. \quad (3.24)$$

For $\gamma = 0$, we get no trade if

$$\pi \leq \frac{r + \kappa}{\xi r + \kappa} \equiv \underline{\pi}. \quad (3.25)$$

Comparing the two thresholds, we obtain that $\underline{\pi} \geq \bar{\pi}$ if and only if $\kappa \geq r$. Finally, for any given π in between these thresholds, the indifference condition requires

$$\pi = \frac{(\kappa + \gamma\lambda)(r + \kappa)}{\kappa(\xi r + \kappa) + \gamma\lambda(\xi\kappa + r)} \quad (3.26)$$

Differentiating we get (up to a constant)

$$\frac{\partial \pi}{\partial \gamma} = (\xi - 1)(r + \kappa)\lambda\kappa(r - \kappa), \quad (3.27)$$

which depends on r relative to κ . In particular, π increases with γ if and only if $r > \kappa$. This gives the following result.

Proposition 1. *For any given $\pi \in (0, 1)$, a steady state equilibrium exists.*

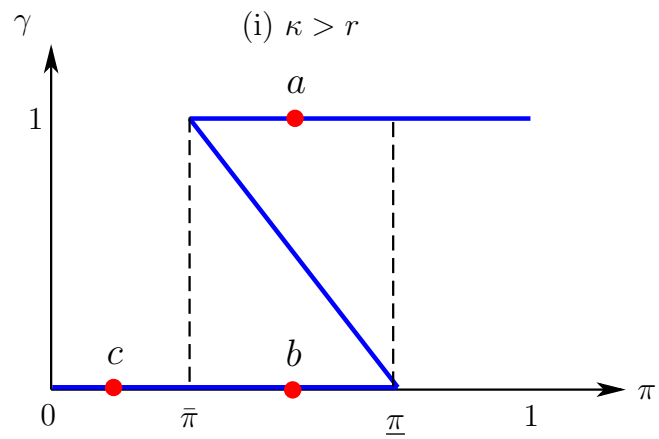
If $\pi \geq \bar{\pi}$, we have that $\gamma = 1$ is a steady state equilibrium in pure strategies, i.e. all buyers trade.

If $\pi \leq \underline{\pi}$, we have that $\gamma = 0$ is a steady state equilibrium in pure strategies, i.e. buyers do not trade.

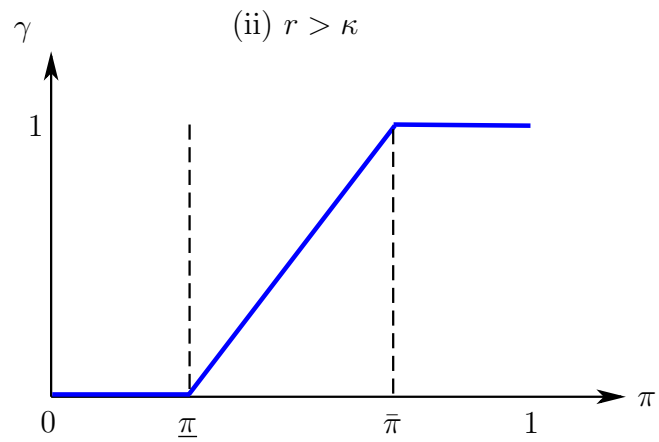
If $\kappa < r$, the steady state equilibrium is unique, with the equilibrium for $\pi \in (\underline{\pi}, \bar{\pi})$ being in mixed strategies.

If $\kappa > r$, for $\pi \in (\bar{\pi}, \underline{\pi})$, there are three steady state equilibria including a mixed strategy one.

As shown in Figure 2, an asset market can be frozen due to a change in expectation or a



no trade multiple equilibria trade



no trade mixing trade

Figure 2: Steady State Equilibrium

change in asset quality. For example when $\kappa > r$, a move from point a to point b represents a *self-fulfilling* market freeze as a result of coordination failure between traders, while a move from point a to point c represents a *fundamental* market freeze as a result of an exogenous drop in the asset quality π .

To see the role of market frictions in determining the equilibrium outcome, one can rewrite the trade surplus in steady state as

$$\frac{\tilde{\pi}}{1 - \tilde{\pi}}(v_o - v_s) + (v_\ell - v_s) \geq 0. \quad (3.28)$$

The first term captures the quality effect. Note that it is given by

$$\frac{\pi}{1 - \pi} \frac{\kappa}{\kappa + \lambda}, \quad (3.29)$$

which is decreasing in λ . As market friction goes down (i.e., higher λ), the fraction of good assets available in the market drops, lowering the average quality of assets and the incentive to trade. The last term captures the dynamic strategic complementarity. That is, if agents are more willing to purchase assets in future, then the present value of obtaining a bad asset goes up, increasing a buyer's incentive to purchase an unknown asset now. The strategic complementarity is increasing in λ . The reason is that upon obtaining a bad asset, it is easier to sell it when market friction is low. Overall, the impact of market frictions depends on the relative strength of these two effects.

Figure 3 shows the effect of λ on the steady state equilibrium. When $\kappa > r$, the strategic complementarity dominates the quality effect. In this case, an increase in λ strengthens the strategic complementarity and makes it easier to support a (full or partial) trading equilibrium. Also, due to strategic complementarity, multiple equilibria can arise. When $\kappa < r$, the quality effect dominates. In this case, an increase in λ lowers the average quality of assets and makes it harder to support a full trade equilibrium.

3.3 Unanticipated Quality Shock and Intervention

We now consider a market freeze induced by a permanent negative shock to the asset quality. Suppose now that the quality in the asset drops unexpectedly at $t = 0$ to a level $\pi(0)$. We assume that $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\}$, so that there is a unique steady state equilibrium of no trade. We show next that after the shock there is convergence to this new steady state with no trade in equilibrium along the path.

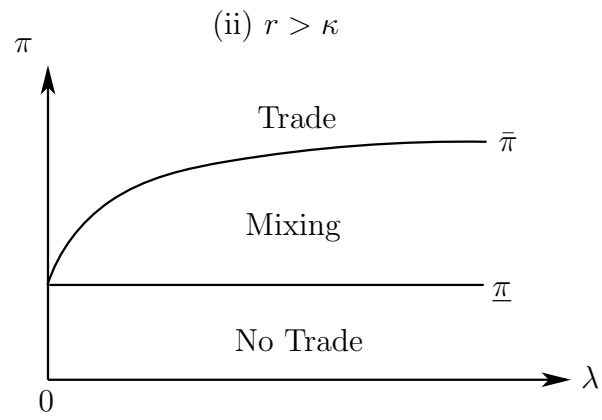
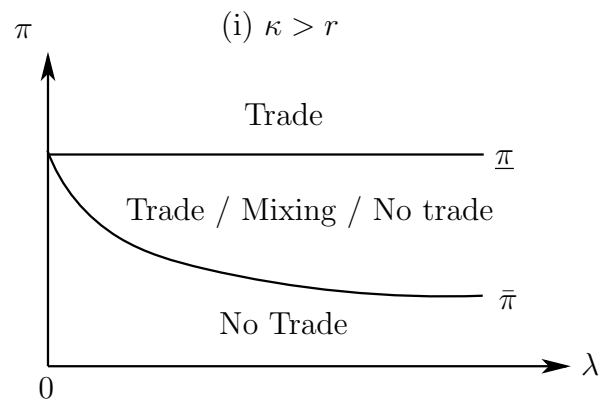


Figure 3: Effect of Market Frictions

Proposition 2. *Suppose $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\} \equiv \pi_{\min}$. There exists an equilibrium with no trade at any t that converges to a steady state without trade. In addition, if $\pi(0) \leq \frac{\kappa\pi_{\min}}{\kappa+(1-\pi_{\min})\lambda}$, this equilibrium is unique.*

Proof. It is clear that there is a unique steady state with no trade for $\pi(0) < \pi_{\min}$, since with no trade in steady state, it must be the case that $\tilde{\pi}(0) = \pi(0)$.

Suppose now there is no trade for any t . The law of motion is then given by

$$\dot{\mu}_s(t) = -\dot{\mu}_o(t) = \kappa\mu_o(t) \quad (3.30)$$

with $\mu_\ell = (1 - \pi(0))S$. This implies that the fraction of good assets on the market for sale at time t ,

$$\tilde{\pi}(t) = \frac{\mu_s(t)}{\mu_s(t) + \mu_\ell(t)}, \quad (3.31)$$

is increasing monotonically to $\pi(0)$. Hence, the asset quality reaches at most $\pi(0)$ over time, a level where there is no trade in steady state. Since $v_\ell(t) = 0$ for all t , we are left to verify that

$$\tilde{\pi}(t)v_o - v_s \leq 0. \quad (3.32)$$

for all t . We have that $\tilde{\pi}(t) < \pi(0)$ for all t . Hence, it suffices to show that

$$\pi(0)v_o - v_s \leq 0. \quad (3.33)$$

We have that

$$\pi(0) \left(\xi \frac{r}{r + \kappa} + \frac{\kappa}{r + \kappa} \right) \leq 1 \quad (3.34)$$

or $\pi(0) \leq \underline{\pi}$. This confirms that no buyer will buy the asset along the transition path if there is no trade at any point in time.

For the last statement, the condition implies that $\max_t \tilde{\pi}(t) < \pi_{\min}$. Then, by the definition of $\pi_{\min}$, it must be the case that

$$\pi(0)(v_o - v_\ell) + (v_\ell - v_s) \leq 0, \quad (3.35)$$

with $v_\ell = \frac{\lambda}{\lambda+r}v_s$. Hence, even if the average quality is at the maximum level, and there is trade ($\gamma(t) = 1$) for all t afterwards, it is not optimal to buy an asset at any t . This completes the proof. $\square$

Without an intervention by a MMLR, the market will freeze forever after a sufficiently large

shock to the quality of the assets. However, removing a sufficient amount of bad assets, ensures that there is a steady state with trade.¹⁷ Let I be the number of assets bought by the MMLR. For $\kappa > r$, we have immediately that there will be trade in steady state as long as

$$\frac{\pi(0)S}{S-I} \geq \bar{\pi} \quad (3.36)$$

or

$$I \geq S \left(1 - \frac{\pi(0)}{\bar{\pi}} \right) \equiv I_{\min}. \quad (3.37)$$

For $\kappa < r$, the issue arises on what level the market functions again, as there exist unique steady state equilibria for all $\gamma \in [0, 1]$. In what follows, we assume that I is such that it satisfies inequality (3.37).¹⁸

3.4 Continuous Trade after Intervention

We first look at how the average quality of the asset changes after the intervention. In general, when there is no trade the average quality of the asset improves as more owners suffer a liquidity shock (switch to x_s) and want to sell, while the number of lemons in the market stays constant. We can also show that – when there is trade after the intervention – the average quality of the asset declines monotonically to the one in the steady state condition we found earlier. Importantly, this is independent of any history of trade before having continuous trade.

Lemma 3. *After the intervention, the average quality of the asset $\tilde{\pi}(t)$ decreases over time whenever there is trade and increases whenever there is no trade.*

Proof. See Appendix. □

The next result establishes that after the intervention at T the average quality of the asset is always above $\tilde{\pi}$. This is the critical level for the average quality of assets being traded so that there is a steady state with trade ($\gamma = 1$). The intuition is as follows. First, without

¹⁷Note that a market can switch from a full trade equilibrium to a no trade equilibrium as a result of a small π shock. For the same reason, the market can be restored by a small intervention.

¹⁸ This allows us to not distinguish between the two cases. In the second case, it could be optimal to only get the market functioning partially again. This would require an intervention that restores a mixed strategy equilibrium with partial trading in steady state.

intervening the floor for the average asset quality is simply given by $\tilde{\pi}(0)$. Prior to the intervention, if there is no trade, the asset quality increases. If at some stage before T , there has been trade, it must be the case that $\mu_s(t) \geq \mu_s(0)$ for all $t < T$. Second, the MMLR removes only bad assets which causes a discrete jump in the average quality at time T , that is sufficient to raise floor for the average asset quality to $\tilde{\pi}$. We denote the quality of assets right after the intervention at T by $\tilde{\pi}(T^+)$ and its long-run steady state level by $\tilde{\pi}(I)$.

Lemma 4. *Suppose the intervention $I \geq I_{\min}$ takes place at T . We have $\tilde{\pi}(T^+) \geq \tilde{\pi}$. Furthermore, $\tilde{\pi}(t)$ converges monotonically to $\tilde{\pi}(I) \geq \tilde{\pi}$ from above if and only if there is trade for some $[t, \infty)$.*

Proof. A lower bound on $\mu_s(t)$ is given by $\frac{\kappa}{\kappa+\lambda}S\pi(0)$. Using the intervention being at least equal to $I = S(1 - \frac{\pi(0)}{\tilde{\pi}})$, this yields a minimum quality just after the intervention equal to

$$\tilde{\pi}(T^+) = \frac{\frac{\kappa}{\kappa+\lambda}S\pi(0)}{\frac{\kappa}{\kappa+\lambda}S\pi(0) + S(1 - \pi(0)) - I} = \frac{\kappa\tilde{\pi}}{\kappa + (1 - \lambda)\tilde{\pi}} = \tilde{\pi}. \quad (3.38)$$

By the previous lemma, $\tilde{\pi}(t)$ will decrease monotonically on this interval for any $t \geq T$ if and only if there is trade in an interval $[t, \infty)$. Also, $\tilde{\pi}(t) \geq \tilde{\pi}(I)$. For $t \rightarrow \infty$, it is straightforward to verify that $\tilde{\pi}(t) \rightarrow \tilde{\pi}(I) \geq \frac{\kappa\tilde{\pi}}{\kappa + (1 - \tilde{\pi})\lambda}$. $\square$

We now show that continuous trade after the intervention is indeed an equilibrium. To ensure trade in the new steady state, the MMLR has to purchase a sufficient amount of the bad asset such that $\tilde{\pi} \geq \tilde{\pi}$ at T . As long as there is trade for any $t \in [T, \infty)$, the economy will converge monotonically to the steady state with trade. We then guess that there is trade after T and verify that buyers have an incentive to buy for any $t \in [T, \infty)$ given that there is trade at any later stage. The intuition is that waiting makes a potential buyer always worse off. First, there is a probability less than one to transact in any future finite time interval. Second, the average quality of the asset declines over time when there is trade.

To establish this result formally, it is convenient to define the function $\Gamma : [T, \infty) \rightarrow \mathbb{R}$ by

$$\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s \quad (3.39)$$

which is the net surplus from buying the asset at t . Note that in this function only the average quality and the value of buying a lemon are time dependent. Also, $\Gamma(t) > 0$ if $\gamma(s) = 1$ for all $s \in (t, \infty)$, as v_ℓ is a constant equal to $\frac{\lambda}{\lambda+r}v_s$. Finally, if there is trade at all future dates, $\tilde{\pi}(t)$ will decrease over time and so does $\Gamma(t)$ as $\tilde{\pi}(t) \geq \tilde{\pi}$.

Define $T_{sup} = \sup\{t > T | v_b(t) > \Gamma(t)\}$. That is T_{sup} is the last time when a buyer does not prefer to buy the asset given that everyone else trades at all $t > T$. Note that we require that in steady state there is trade. This implies that $T_{sup} < \infty$, as otherwise there cannot be convergence to the steady state. We are left to show that at any time $t \in [T, \infty)$ buyers do not prefer to delay the purchase of the asset.

Proposition 5. *Continuous trade after the intervention is an equilibrium.*

Proof. Since both Γ and v_b are continuous over the interval $[T, \infty)$, it suffices to show that $\Gamma(T_{sup}) > v_b(T_{sup})$. By definition of T_{sup} , $\Gamma(t) \geq v_b(t)$ for all $t \in [T_{sup}, \infty)$.

Consider some small time interval of size Δ . Let $\bar{\mu} = \max_{t \in [T, \infty)} \mu_s(t) + \mu_\ell$. We have,

$$v_b(T_{sup}) \leq \frac{1}{1+r\Delta} \left(\lambda \bar{\mu} \Delta \Gamma(T_{sup} + \Delta) + (1 - \lambda \bar{\mu} \Delta - \kappa \Delta) v_b(T_{sup} + \Delta) + o(\Delta) \right) \quad (3.40)$$

$$\leq \frac{1}{1+r\Delta} \left((1 - \kappa \Delta) \Gamma(T_{sup} + \Delta) + o(\Delta) \right), \quad (3.41)$$

where the inequalities follow from the fact that $\Gamma(T_{sup} + \Delta) \geq v_b(T_{sup} + \Delta)$. Hence, it suffices to show that

$$\Gamma(T_{sup}) > \frac{1}{1+r\Delta} \left((1 - \kappa \Delta) \Gamma(T_{sup} + \Delta) + o(\Delta) \right). \quad (3.42)$$

Rearranging, we obtain

$$r\Gamma(T_{sup}) + \kappa\Gamma(T_{sup} + \Delta) > \frac{\Gamma(T_{sup} + \Delta) - \Gamma(T_{sup})}{\Delta} + \frac{o(\Delta)}{\Delta}. \quad (3.43)$$

The first term on the right-hand side is strictly negative as $\dot{\Gamma}(t) < 0$ for all $t \in (T_{sup}, \infty)$ by the definition of T_{sup} . Hence, we only need to show that

$$r\Gamma(T_{sup}) + \kappa\Gamma(T_{sup} + \Delta) > \frac{o(\Delta)}{\Delta}. \quad (3.44)$$

The left-hand side is strictly positive, as $\Gamma(t) > 0$ for all $t \in [T, \infty)$. Hence, for $\Delta \rightarrow 0$ we obtain $(r + \kappa)\Gamma(T_{sup}) > 0$, which completes the proof. $\square$

4 Intervention and Market Recovery

From now on, we assume that there is a shock to the quality of the asset at $t = 0$ such that $\pi(0) < \bar{\pi}$. The MMLR can make an announcement at $t = 0$ when the shock is realized

about his policy. The policy consists of announcing a time $T \geq 0$ to buy I assets at price p_M . It is important that the MMLR can commit to his announcement. We study now when the market will function again given the announcement of T . Note that trading can resume prior to the actual intervention. The exact trading pattern before T will determine $\tilde{\pi}(T^+)$, and, hence, the exact transition to the steady state after the intervention. When the market will function again, depends in turn on the value of the intervention.

To facilitate our analysis slightly, we assume that buyers that decline to trade have a zero probability of trading again. This implies that the outside option for potential buyers is constant at 0 and a buyer will decide to buy if and only if

$$\tilde{\pi}(t)(v_o - v_\ell(t)) + (v_\ell(t) - v_s) \geq 0. \quad (4.1)$$

This assumption removes the option for buyers to wait, but does not change any of the results in the previous sections. We also take it as given that the equilibrium after the intervention is monotone convergence to a steady state with trading, as shown in the previous section.

4.1 The Option Value of the Intervention

The MMLR also makes a take-it-or-leave-it offer for purchasing the asset at T denoted by $p_M(T)$. As discussed in the next section, the MMLR has always an incentive to minimize the costs of intervention, so the price that the MMLR offers will make a lemon just indifferent between waiting and selling immediately at T . Since after the intervention there will always be trade, his value is given by

$$v_\ell(T^+) = \frac{\lambda\mu_b}{\lambda\mu_b + r} \frac{\delta - x_s}{r}, \quad (4.2)$$

where T^+ is the right-hand limit for $t \rightarrow T$. The price the MMLR needs to offer to make a lemon just indifferent between waiting to sell on the market and selling to the MMLR immediately is thus given by $p_M(T) = v_\ell(T^+)$. The market price for trading the asset is unaffected by the intervention. Hence, if there is trade this price is

$$p_m(t) = v_s(t) = (\delta - x_s)/r, \quad (4.3)$$

as good sellers will only sell their asset at their reservation price. Hence, prices in the market are constant over time as long as there is trade.¹⁹

When intervening at the minimum level $I_{\min}$, we obtain that the value of acquiring a lemon just an instant before the intervention is given by

$$v_\ell(T^-) = \frac{I_{\min}}{S(1 - \pi(0))} p_M(T) + \left(1 - \frac{I_{\min}}{S(1 - \pi(0))}\right) \frac{\lambda}{\lambda + r} \left(\frac{\delta - x_s}{r}\right) = \frac{\lambda}{\lambda + r} v_s. \quad (4.4)$$

The reason is that the instantaneous probability of a trade on the market at time T is 0. The first term is thus the expected value of a trade with the MMLR at T . Conditional on not being able to sell to the MMLR, the lemon can still sell on the market after the intervention as there will be trade again. This is the second term. This implies that being a lemon just before the intervention is the same as in a steady state with trading again.

Interestingly, changing the size of the intervention given the minimum price $p_M(T)$ does not increase the option value of the intervention for buyers. However, the MMLR can increase the price for buying assets to any price $\frac{\lambda}{\lambda + r} v_s < p_M(T) \leq v_s$. This increases the option value. Once the MMLR has done so, increasing I can even increase the option value more, as the first term in $v_\ell(T^-)$ is now strictly greater than the second one. Denoting the value of the option by V_I , we then have that

$$0 \leq V_I \leq \frac{r}{r + \lambda} v_s \quad (4.5)$$

with the value for a lemon given by

$$v_\ell(T^-) = \frac{\lambda}{\lambda + r} v_s + V_I. \quad (4.6)$$

Note that at T there is a discrete drop in the value due to the activity by the MMLR if we have $V_I > 0$.²⁰ Prior to the intervention, a lemon will always prefer to sell the asset if he encounters a trade opportunity where the buyer wants to buy the asset. If he does not have a trading opportunity, he will still be a lemon at the time of the intervention T . This yields a value function for a lemon at $t < T$ equal to

$$v_\ell(t) = E_t \left[e^{-r(\tau_m - t)} p_m \mathbf{1}_{\{\tau_m < T\}} + e^{-r(T - t)} (p_M(T) + V_I) \mathbf{1}_{\{\tau_m \geq T\}} \right], \quad (4.7)$$

where τ_m is the random time for the next trade opportunity provided the market functions

¹⁹In other words, the intervention will not affect the prices in the market. But this is an artefact of our assumption that buyers have all the bargaining power.

²⁰Note that this function is left-continuous at T , as it expresses the value of being a lemon at this time having the option to transact with the MMLR.

again. Solving for $\gamma(t) = 1$ in some interval $[t, T]$ we get

$$v_\ell(t) = \frac{\lambda}{\lambda + r} \left(\frac{\delta - x_s}{r} \right) [1 - e^{-(r+\lambda)(T-t)}] + v_\ell(T^-) e^{-(r+\lambda)(T-t)} \quad (4.8)$$

or

$$v_\ell(t) = \frac{\lambda}{\lambda + r} \left(\frac{\delta - x_s}{r} \right) + V_I e^{-(r+\lambda)(T-t)}. \quad (4.9)$$

If there is no trade in the interval $[t, T]$, we simply have $v_\ell(t) = v_\ell(T^-) e^{-r(T-t)}$. Finally, note that with partial trading ($\gamma(t) \in (0, 1)$) we do not have a closed form solution. However, $v_\ell(t) \leq v_\ell(T^-)$.

The value of buying a lemon depends therefore on the intervention itself as well as how the market reacts to the announcement of there being an intervention at T . Being able to sell to the MMLR is an option value. This option value increases, if such an intervention resurrects the market already prior to the intervention. But it is lower due to discounting, the earlier the investor trades before the intervention.

4.2 The Structure of Equilibria Before T

We now characterize equilibria prior to the intervention of the MMLR at T . It is again useful to define the expected value of buying the asset. The function $\Gamma(t) : [0, T) \rightarrow \mathbb{R}$ depends now on the fact whether there is trade or not, which drives both $v_\ell(t)$ and $\tilde{\pi}(t)$. It is again defined as

$$\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s. \quad (4.10)$$

Note that this function is continuous on $[0, T)$, but can jump at T , as the average quality increases at this point and with $V_I > 0$ the value of a lemon decreases. Furthermore, it is right-continuous at T , as buying at T gives no chance to transact with the MMLR. The direction of the jump, however, is not clear. The fraction of good asset increases, while the value of being a lemon decreases. First, we show that once $\Gamma(t)$ becomes positive it has to remain there.

Lemma 6. *If $\Gamma(t_0) \geq 0$ for some $t_0 < T$, then $\Gamma(t_1) \geq 0$ for all $t_1 \in (t_0, T)$.*

Proof. Suppose not. Then, there exists a $t_1 \in (t_0, T)$ such that $\Gamma(t_1) < 0$. As Γ is continuous, this implies that there must be an interval $(\tau_0, \tau_1) \subset (t_0, t_1)$ where there is no trade, i.e. $\gamma(t) = 0$. But then, over this interval, the average quality $\tilde{\pi}(t)$ increases and we have that

$\dot{v}_\ell(t) = v_\ell(T^-)e^{-r(T-t)}r > 0$. Hence, $\Gamma(t)$ must be strictly increasing over this interval starting out at $\Gamma(\tau_0) = 0$. A contradiction. $\square$

This implies immediately that after we have some trade ($\gamma(t) > 0$), we cannot go back to no trading anymore. It turns out that we can strengthen the result in such a fashion that once the market fully recovers before the intervention at T , it will remain there in equilibrium.

Lemma 7. *If $\gamma(t) = 1$ for some interval $[t_0, t_1]$ with $t_1 < T$, then $\Gamma(t)$ is strictly convex over this interval.*

Proof. We have

$$\Gamma(t) = \tilde{\pi}(t)(v_o - v_\ell(t)) + (v_\ell(t) - v_s) \quad (4.11)$$

$$\dot{\Gamma}(t) = \dot{\tilde{\pi}}(t)(v_o - v_\ell(t)) + (1 - \tilde{\pi}(t))\dot{v}_\ell(t) \quad (4.12)$$

$$\ddot{\Gamma}(t) = \ddot{\tilde{\pi}}(t)(v_o - v_\ell(t)) - 2\dot{\tilde{\pi}}(t)\dot{v}_\ell(t) + (1 - \tilde{\pi}(t))\ddot{v}_\ell(t). \quad (4.13)$$

We will show that $\Gamma(t)$ is strictly convex if it is positive and strictly increasing. Assuming $\gamma(t) = 1$ for the asset quality and omitting time indexes, we have

$$\dot{\tilde{\pi}} = (1 - \tilde{\pi})\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} \quad (4.14)$$

$$\ddot{\tilde{\pi}} = -\dot{\tilde{\pi}}\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + (1 - \tilde{\pi})\frac{(\mu_s + \mu_\ell)\ddot{\mu}_s - (\dot{\mu}_s)^2}{(\mu_s + \mu_\ell)^2}. \quad (4.15)$$

Since $\gamma(t) = 1$, $\dot{\tilde{\pi}}(t) < 0$. Also, we either have continuous trade or we need to have a non-trade or mixing region before. Assume w.l.o.g. that $\Gamma(t)$ crosses zero from below at time t_0 . Hence, at t_0 , we need that the right-hand derivative $\dot{\Gamma}(t_0^+) > 0$. This can only be the case if $\dot{v}_\ell(t_0^+) > 0$. Also, we have that

$$v_\ell(t) = \frac{\lambda}{\lambda + r}v_s(1 - e^{-(r+\lambda)(t_1-t)}) + v_\ell(t_1)e^{-(r+\lambda)(t_1-t)}. \quad (4.16)$$

Hence, $\dot{v}_\ell(t) > 0$ for $t \in [t_0, t_1]$ if and only if $\dot{v}_\ell(t_0^+) > 0$. This implies that $v_\ell(t)$ is a strictly increasing and strictly convex function over this interval. Hence, the last two terms are positive in the expression for $\ddot{\Gamma}(t)$. Note that $v_o - v_\ell(t) > 0$. If $\ddot{\tilde{\pi}}(t)$ is positive we are done.

Suppose to the contrary that $\ddot{\tilde{\pi}}(t) < 0$. As long as $\dot{\Gamma}(t) > 0$ over the interval, it must be the case that

$$0 < v_o - v_\ell(t) < -\frac{1 - \tilde{\pi}(t)}{\dot{\tilde{\pi}}(t)}\dot{v}_\ell(t). \quad (4.17)$$

Using this in the expression for $\ddot{\Gamma}(t)$, it suffices to show that

$$-\ddot{\pi}(t)\frac{1-\tilde{\pi}(t)}{\dot{\pi}(t)}\dot{v}_\ell(t) - 2\dot{\pi}(t)\dot{v}_\ell(t) + (1-\tilde{\pi}(t))\ddot{v}_\ell(t) > 0 \quad (4.18)$$

or that

$$-\frac{\ddot{\pi}(t)}{\dot{\pi}(t)} - 2\frac{\dot{\pi}(t)}{1-\tilde{\pi}(t)} + \frac{\ddot{v}_\ell(t)}{\dot{v}_\ell(t)} > 0. \quad (4.19)$$

Note that $\ddot{v}_\ell(t) = (r + \lambda)\dot{v}_\ell(t)$. Hence, rewriting and using the fact that $\ddot{\mu}_s = -(\kappa + \lambda)\dot{\mu}_s$, we obtain

$$-\left[-\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + \frac{-(\mu_s + \mu_\ell)(\kappa + \lambda) - \dot{\mu}_s}{\mu_s + \mu_\ell}\right] - 2\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + (\lambda + r) > 0 \quad (4.20)$$

$$(\kappa + \lambda) + (r + \lambda) > 0 \quad (4.21)$$

which completes the proof. $\square$

This implies that we have fully characterized all possible equilibria.

Proposition 8. *All equilibria before T can be characterized by two breaking points $\tau_1(T) \geq 0$ and $\tau_2(T) \in [\tau_1, T)$ such that*

- (i) *there is no trade ($\gamma(t) = 0$) in the interval $[0, \tau_1)$,*
- (ii) *there is partial trade ($\gamma(t) \in (0, 1)$) in the interval $[\tau_1, \tau_2)$,*
- (iii) *there is full trade ($\gamma(t) = 1$) in the interval $[\tau_2, T)$.*

The two breaking points cannot be determined analytically, unless in the special case where $\tau_1 = \tau_2$. However, we can characterize when there is some recovery in the market before the intervention. We turn to analyzing such an *announcement* effect next.

4.3 Announcement Effect: Partial and Full Recovery Before T

In general, there might be multiple equilibria due to the (intertemporal) strategic complementarity of trading the asset. However, we will focus mostly on equilibria where trading resumes before the intervention. It turns out that such scenarios can be described nicely by the equilibrium value for the surplus function from trading, $\Gamma(t)$. We first characterize when the market can recover before T .

Proposition 9. Denote $\tilde{\pi}_{max}(T^-)$ the average asset quality at T before the intervention conditional on there being no trade in $[0, T)$. There does not exist an equilibrium with trade before the intervention at T ($\tau_1(T) = \tau_2(T) = T$) if and only if

$$\tilde{\pi}_{max}(T^-)v_o + (1 - \tilde{\pi}_{max}(T^-)) \left(\frac{\lambda}{\lambda + r} v_s + V_I \right) - v_s \leq 0. \quad (4.22)$$

Proof. Suppose the condition is satisfied. The surplus from buying the asset at $t \in [0, T)$ is given by

$$\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s \quad (4.23)$$

$$< \tilde{\pi}_{max}(T^-)v_o + (1 - \tilde{\pi}_{max}(T^-))v_\ell(t) - v_s \quad (4.24)$$

$$\leq \tilde{\pi}_{max}(T^-)v_o + (1 - \tilde{\pi}_{max}(T^-))v_\ell(T^-) - v_s \quad (4.25)$$

$$= \tilde{\pi}_{max}(T^-)v_o + (1 - \tilde{\pi}_{max}(T^-)) \left(\frac{\lambda}{\lambda + r} v_s + V_I \right) - v_s \quad (4.26)$$

$$\leq 0, \quad (4.27)$$

since $v_\ell(t) \leq v_\ell(T^-)$. Hence, $\Gamma(t) < 0$ for all $t < T$ independent of the trading strategy $\gamma(t)$. This implies $\gamma(t) = 0$ is the unique equilibrium for all $t \in [0, T)$.

Suppose now the condition is not satisfied, but that $\gamma(t) = 0$ for all $t \in [0, T)$ is an equilibrium. We have that $\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t)) \left(\frac{\lambda}{\lambda + r} v_s + V_I \right) e^{-r(T-t)} - v_s$. As $t \rightarrow T$, $\Gamma(t) \rightarrow \Gamma(T^-) > 0$. Hence, even if no one trades before the intervention, it is optimal to buy the asset for t sufficiently close to T . This implies that $\gamma(t) = 0$ for all $t \in [0, T)$ cannot be an equilibrium, a contradiction. Hence, there must exist an equilibrium with $\tau_1 < T$. $\square$

This result has important implications. Whether there can be a recovery in the market depends on the size of the shock ($\pi(0)$) and the policy of the MMLR. Note that we can rewrite equation (4.22) to obtain a sufficient and necessary condition for recovery given by

$$\tilde{\pi}_{max}(T^-) \leq h(V_I) \quad (4.28)$$

where h is a threshold that decreases with the policy variable V_I . The maximum average quality, however, increases with T , but decreases with the shock $\pi(0)$. Furthermore, it is bounded by the initial level of quality $\pi(0)$. Hence, for given T , one has to increase the option value V_I sufficiently to induce trading before the intervention. Also, provided the shock is not too large, if T increases, there will be at least a partial recovery in the market

prior to T , as the quality of the asset increases when the market is frozen. In particular, we have the following result.

Corollary 10. *Suppose $V_I = 0$. There cannot be trade with $\gamma(t) = 1$ for any interval before T (i.e. $\tau_2(T) = T$). Furthermore, if $\pi(0) \leq \frac{\kappa\pi_{\min}}{\kappa+(1-\pi_{\min})\lambda}$, the unique equilibrium before the intervention is no trade for any T (i.e. $\tau_1(T) = T$).*

Proof. Suppose that $0 \leq \tau_2(T) < T$. Then, $\tilde{\pi}(t)$ declines over $[\tau_2, T]$, while $v_\ell(t) = v_\ell(T)$. Hence, the surplus from trade $\Gamma(t)$ declines as well over this interval. Also, it must be the case that $\gamma(t) < 1$ for $t \in [0, \tau_2)$ given the initial shock $\pi(0) < \pi_{\min}$. Hence, $\Gamma(t) \leq 0$ for $[0, \tau_2)$. Since $\Gamma(t)$ must be continuous over $[0, T)$, this implies that $\Gamma(t) < 0$ for $(\tau_2, T]$. A contradiction.

For the second part, note that the surplus for buyers is given by

$$\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s \quad (4.29)$$

$$< \pi(0)v_o + (1 - \pi(0))v_\ell(T) - v_s \quad (4.30)$$

$$\leq \tilde{\pi}_{\min}v_o - (1 - \tilde{\pi}_{\min})v_\ell - v_s \quad (4.31)$$

$$= 0. \quad (4.32)$$

Hence, the surplus from trading is strictly negative for any $t_0 < T$ prior to the intervention, even if there is trade with $\gamma(t) = 1$ for all $T > t > t_0$. Hence, it is a strictly dominant strategy to not trade. $\square$

4.4 Continuous Trade

A special case arises, when an announcement leads to continuous (full) trade (i.e. $\tau_2(T) = 0$). By the previous result, this can only be the case if the option value V_I is large enough for a given T . With continuous trade in $[0, \infty)$, the economy jumps immediately to a new steady state at $t = 0$, with only the number of sellers with bad assets dropping at the time of the intervention T . This implies that one can characterize continuous trade by only looking at the limit of the expected value of buying the security at $t = 0$.

Proposition 11. *There exists an equilibrium with continuous trade, if and only if*

$$\tilde{\pi}(0)v_o + (1 - \tilde{\pi}(0)) \left(\frac{\lambda}{\lambda + r} v_s + V_I e^{-(r+\lambda)T} \right) - v_s \geq 0 \quad (4.33)$$

Proof. If the condition is satisfied, we have that

$$\Gamma(t) > \Gamma(0) \geq 0 \quad (4.34)$$

for all $t \in [0, T]$, as $v_\ell(t)$ is strictly increasing when there is trading for all t . If the condition does not hold, it is optimal to not trade for t sufficiently close to 0, even if there is trade forever afterwards. $\square$

Since $v_o > v_s$, it is always feasible for the MMLR to delay the intervention, but to induce continuous trade by setting V_I sufficiently high. The intuition is simple. If the MMLR offers v_s for sure, it completely insures investors for buying a lemon just before the intervention. Hence, there will be trade at this stage. Of course, the option value at $t = 0$ decreases when the intervention is further away in the future.

Totally differentiating condition (4.33), we obtain

$$\frac{dT}{dV_I} = \frac{1}{(r + \lambda)V_I} > 0 \quad (4.35)$$

for $\frac{r}{r+\lambda}v_s > V_I > V_I^{\min} > 0$. Here $V_I^{\min}$ is defined as the minimum option value needed to be able to support continuous trade with $T > 0$, or

$$V_I^{\min} = \frac{\tilde{\pi}(0)}{1 - \tilde{\pi}(0)}(v_s - v_o) + \frac{r}{r + \lambda}v_s. \quad (4.36)$$

Hence, we have that a given intervention of size V_I allows for continuous trade as long as

$$T < T^c = \frac{1}{r + \lambda} \ln \left(\frac{V_I}{V_I^{\min}} \right). \quad (4.37)$$

It is useful to summarize the different cases for market recovery before the intervention. Figure 4 depicts the situation of market recovery for different values for the shock $\pi(0)$ and the option value V_I *independent of the time of intervention*. Hence, independent of the shock, there always exists a sufficiently high option value so that the MMLR could postpone the intervention, but still achieve continuous trade (i.e., $\tau_2(T) = 0$ for some $T > 0$). Conversely, if the option value V_I is sufficiently low, then there is never trade prior to the intervention for any given shock $\pi(0) < \tilde{\pi}_{\min}$ (i.e., $\tau_1 = T$). In the middle region, we may have an announcement effect for some $T > 0$.

Figure 5 shows the same situation for a given value $T > 0$. The dashed lines depict the

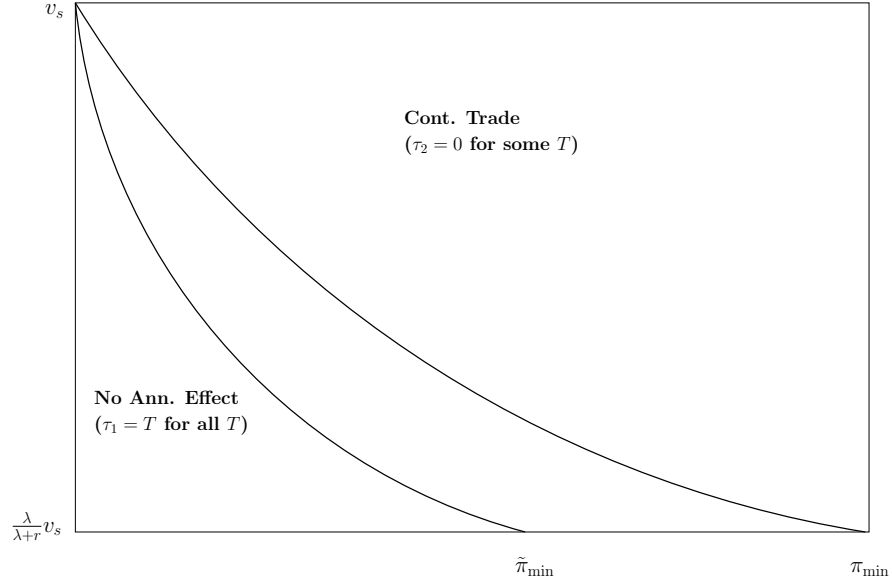


Figure 4: Announcement Effects independent of T in $(\pi(0), V_I)$ -space

boundaries of the areas of Figure 4. Observe that the larger is T , the higher V_I has to be to achieve continuous trade. As T increases, the solid line shifts further to the right, implying that the option value has to increase for any given shock to ensure continuous trade. Eventually, T becomes too large to ensure continuous trade altogether.

If the intervention takes place at some T , the quality of the asset in the market might not improve sufficiently for people to trade prior to the intervention. Hence, as T increases, the region where there is an announcement effect becomes larger, eventually converging to the dashed line. Hence, the middle region – where $0 < \tau_1 < T$ – tends to expand as T increases.

Finally, note that for T sufficiently close to 0 our results imply a bang-bang result with respect to the equilibrium. In other words, at $T = 0$ the lower solid line in Figure 5 coincides with the upper dashed line. The value of buying an asset at 0 for $T \rightarrow 0$ when the option value is V_I is given by

$$\Gamma(0^-) \equiv \tilde{\pi}(0)v_o + (1 - \tilde{\pi}(0)) \left(\frac{\lambda}{\lambda + r} v_s + V_I \right) - v_s. \quad (4.38)$$

When $\Gamma(0^-)$ is negative then there exists a unique no trade equilibrium for T sufficiently close to 0. This depends again purely on the size of the option value V_I for any given quality $\pi(0)$.

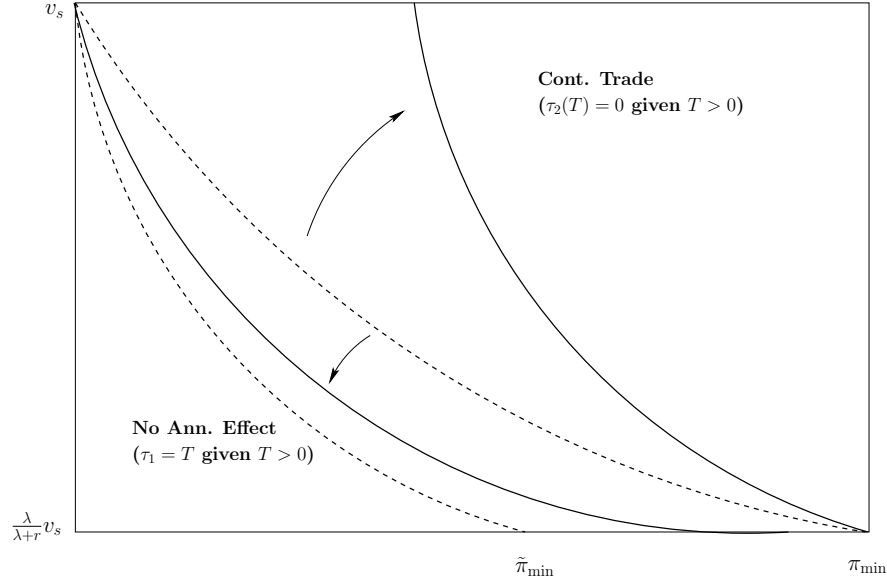


Figure 5: Announcement Effects for a given $T > 0$ in $(\pi(0), V_I)$ -space

4.5 Numerical Examples

This section presents some numerical examples illustrating the effects of policy and key parameters on the trading dynamics. Table 1 gives parameter values for our numerical example. Note that in this example, we have $\kappa > r$ and hence $\pi_{\min} = \bar{\pi}$.

κ	r	λ	$\bar{\pi}$	x_s	x_b
2	0.5	4	0.45	0.5	0.5

Table 1: Parameter values

Suppose there is a quality shock that drives π to $\pi_0 = 0.30$ and thus there is a permanent market freeze. The MMLR can resurrect the market by choosing intervention policy T and V_I . Figure 6 illustrates the effects of intervention on the timing of recovery (τ_1, τ_2) .

- (i) Fix $V_I = 0.03$. As the MMLR chooses a later intervention time T , the timing of partial and full recovery, τ_1 and τ_2 , are delayed. So there is monotonicity of recovery times in T . Furthermore, announcement effect is present only for sufficiently high T . This is because, when V_I is not too high as in this example, it takes time for the market to accumulate enough selling pressure to significantly improve the average quality of asset in the market to induce trade before intervention.
- (ii) Fix $T = 0.3$. Recovery time is monotonically decreasing in the option value V_I .

Moreover, for large V_I , there is no transition period with partial recovery. For V_I sufficiently large, there is continuous trade.

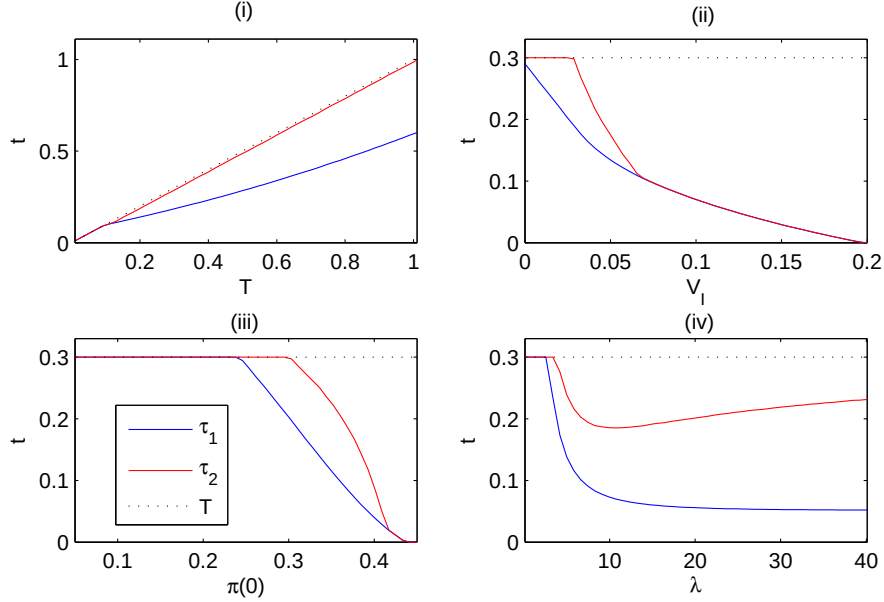


Figure 6: Time of recovery as a function of T , V_I , $\pi(0)$, λ

Now, fixing the intervention policy at $V_I = 0.03$ and $T = 0.3$, Figures (iii)-(iv) study how the policy effect is affected by the sizes of quality shock ($\pi(0)$) and market friction (λ).

- (iii) As the size of the quality shock drops (i.e., higher $\pi(0)$), the announcement effect is monotonically increasing. For $\pi(0)$ sufficiently close to $\pi_{\min}$, there is continuous trade.
- (iv) The effect of the market friction parameter, λ , is non-monotonic. On the positive side, an increase in λ makes the resale of a bad asset easier and hence increasing buyers' incentive to purchase. On the negative side, an increase in λ reduces the buyers' incentive to purchase for two reasons. First, it reduces market frictions and thus reduces the fraction of good assets available in the market, lowering the average quality of assets. Second, it reduces the probability that a buyer, after having purchased a bad asset, will resell it to the MMLR and receive V_I (i.e., a lemon is more likely to be resold before T). This will reduce the discounted option value of the intervention, $V_I e^{-(r+\lambda)T}$.

5 Optimal Time and Size of Intervention

5.1 Objective Function of the MMLR

The objective of the MMLR is a weighted average of the investors' payoffs from holding the security and the costs for MMLR from buying assets. Due to linear utility, we have for any time interval

$$(1 - \theta) \int [(\mu_o(t) + \mu_s(t))\delta + \mu_o(t)x_b - \mu_s(t)x_s]e^{-rt}dt + (1 - \theta) \int Tr(t)e^{-rt}dt + \theta \int (-1)Tr(t)e^{-rt}dt, \quad (5.1)$$

where θ is the weight on the MMLR's costs. Note that $Tr(t)$ are *total* transfers to investors in acquiring the bad assets which net to 0. The first term measures the net benefit of holding assets by the investors. The second term is the transfers received by the investors, while the third expresses the transfers paid by the MMLR. Rewriting, we obtain

$$(1 - \theta) \left[\pi S \delta \int e^{-rt}dt + \int (\mu_o(t)x_b - \mu_s(t)x_s)e^{-rt}dt \right] - (2\theta - 1) \int Tr(t)e^{-rt}dt, \quad (5.2)$$

where $Tr(T) = p_M(T)I$ and 0 otherwise.

We need to restrict θ . For $\theta < 1/2$, the planner would simply maximizing transfers into the economy. Hence, $\theta \in [1/2, 1]$. We can then think of θ as a measure for the importance of resurrecting a market. Importantly, we only measure welfare for the economy according to the allocation of assets across investors. The social value of getting the market working again is given by how the assets with positive dividend are distributed across investors.

5.2 No Intervention in Normal Times

First, we show that intervention is not optimal when there is no shock that pushes the average quality of assets below $\min\{\underline{\pi}, \bar{\pi}\}$. Hence, we confirm that the MMLR should be active only in the event of a bad enough shock.

Proposition 12. *Suppose there is continuous trade. Buying $I > 0$ bad assets is not welfare-improving.*

Proof. The welfare in (5.2) can be rewritten (up to some constant terms) as

$$(1 - \theta)(x_s + x_b) \int_{t=0}^{\infty} \mu_o(t) e^{-rt} dt - e^{-rT} (2\theta - 1) p_M(T) I. \quad (5.3)$$

When there is continuous trade, an intervention does not affect $\mu_o(t)$, but incurs the cost of intervention (i.e. the second term) which is negative for all $\theta > 1/2$. $\square$

The intuition for this result is straightforward. Removing more bad asset will not influence trading behavior, as long as new investors have an incentive to trade. With $\theta > 1/2$ transfers are, however, costly. We next investigate the optimal intervention for the special case of continuous trade.

5.3 Special Case: Ensuring Continuous Trade

Suppose now we require the MMLR to have continuous trade. From the previous section, we know that independent of the size of the shock, it is always feasible to have continuous trade with $T > 0$ as long as the size of the intervention is large enough. In this case, the optimal intervention size V_I^* simply minimizes the costs of the intervention, as there are no costs – beyond the search frictions – from a lack of trading in the market. Hence, this case is interesting to study, as it looks at policies that minimize the costs for the MMLR.

When the MMLR delays the intervention, he must necessarily set $V_I > V_I^{\min} > 0$. The additional option value per lemon can be expressed as

$$\begin{aligned} V_I &= \frac{I_{\min} + \Delta_I}{S(1 - \pi(0))} (p_M(T) + \Delta_p) + \left(1 - \frac{I_{\min} + \Delta_I}{S(1 - \pi(0))}\right) p_M(T) - p_M(T) \\ &= \frac{I_{\min} + \Delta_I}{S(1 - \pi(0))} \Delta_p, \end{aligned} \quad (5.4)$$

where $p_M(T) = \frac{\lambda}{\lambda + r} v_s$ is the minimum price necessary to purchase lemons during the intervention. The total additional transfer for the MMLR associated with a positive option value is given by $V_I S(1 - \pi(0))$.

The objective function is then given by

$$\begin{aligned} Tr(V_I) e^{-rT^c} &= (p_M(T) I_{\min} + S(1 - \pi(0)) V_I) e^{-rT^c(V_I)} \\ &= \left(\frac{\lambda}{\lambda + r} v_s \frac{1 - \frac{\pi(0)}{\pi_{\min}}}{1 - \pi(0)} + V_I \right) S(1 - \pi(0)) e^{-rT^c(V_I)} \end{aligned} \quad (5.5)$$

where $T^c(V_I)$ is the largest time to have continuous trade given the intervention size V_I and $V_I \geq V_I^{\min}$. Hence, a MMLR would minimize this objective function with respect to V_I . We show next that the MMLR has no incentive to delay the intervention to achieve continuous markets.

Proposition 13. *Suppose the MMLR has to ensure continuous markets. The optimal policy is to intervene immediately ($V_I^* = 0$).*

Proof. Differentiating the objective function with respect to V_I , we obtain

$$S(1 - \pi(0))e^{-rT^c} \frac{\lambda}{\lambda + r} \left(1 - \frac{r}{r + \lambda} \frac{v_s}{V_I} \frac{1 - \frac{\pi(0)}{\pi_{\min}}}{1 - \pi(0)} \right). \quad (5.6)$$

Hence, a candidate solution is given by

$$V_I^* = \frac{r}{r + \lambda} v_s \frac{1 - \frac{\pi(0)}{\pi_{\min}}}{1 - \pi(0)}. \quad (5.7)$$

It is straightforward to verify that the second order derivative is positive at this point. Furthermore, the objective function has a global minimum at V_I^* .

We show next that $V_I^* \leq V_I^{\min}$. Suppose to the contrary that $V_I^* > V_I^{\min}$. Rewriting this condition we obtain

$$\left(\frac{r}{r + \lambda} \right) v_s \left(\frac{\pi(0)}{1 - \pi(0)} \right) \left(1 - \frac{1}{\pi_{\min}} \right) > (v_s - v_o) \left(\frac{\tilde{\pi}(0)}{1 - \tilde{\pi}(0)} \right) \quad (5.8)$$

$$\left(\frac{\kappa + \lambda}{\kappa} \right) \left(1 - \frac{1}{\pi_{\min}} \right) > \left(\frac{v_s - v_o}{v_s} \right) \left(\frac{r + \lambda}{r} \right) \quad (5.9)$$

$$\left(\frac{\kappa + \lambda}{\kappa} \right) \left(1 - \frac{1}{\pi_{\min}} \right) > \left(\frac{r}{r + \kappa} \right) (1 - \xi) \left(\frac{r + \lambda}{r} \right). \quad (5.10)$$

Suppose first that $\kappa > r$. This implies that $\pi_{\min} = \bar{\pi}$. Then,

$$\left(\frac{\kappa + \lambda}{\kappa} \right) \left(\frac{(\kappa + \lambda) - \kappa(\xi r + \kappa) - \lambda(\xi \kappa + r)}{(\kappa + \lambda)(r + \kappa)} \right) > \left(\frac{r}{r + \kappa} \right) (1 - \xi) \left(\frac{r + \lambda}{r} \right) \quad (5.11)$$

$$\frac{1}{\kappa} [\kappa r(1 - \xi) + \lambda \kappa(1 - \xi)] > (1 - \xi)(r + \lambda) \quad (5.12)$$

which is a contradiction.

Now suppose that $\kappa \leq r$ which implies that $\pi_{\min} = \underline{\pi}$. We have now that

$$\left(\frac{\kappa + \lambda}{\kappa}\right) \left(1 - \frac{1}{\pi_{\min}}\right) > \left(\frac{r}{r + \kappa}\right) (1 - \xi) \left(\frac{r + \lambda}{r}\right) \quad (5.13)$$

$$\left(\frac{\kappa + \lambda}{\kappa}\right) \left(\frac{r}{r + \kappa}\right) (1 - \xi) > \left(\frac{r}{r + \kappa}\right) (1 - \xi) \left(\frac{r + \lambda}{r}\right) \quad (5.14)$$

$$\kappa > r, \quad (5.15)$$

since $\xi > 1$. Again a contradiction, as for $r = \kappa$ we have $V_I^* = V_I^{\min}$.

The solution for minimizing the costs when delaying is given by $V_I^{\min}$, as the objective function is strictly increasing for all $V_I > V_I^*$. This implies that delaying the intervention at the expense of a larger intervention cannot be optimal to ensure continuous markets. $\square$

The intuition for this result is that delaying involves a fixed cost, as the MMLR has to pay at least the option value $V_I^{\min}$. Furthermore, differentiating the objective function shows that a MMLR minimizes the costs of a delayed intervention, when he purchases the minimum amount of assets, $I_{\min}$, at the maximum price v_s . The option value associated with this policy, V_I^* , is below $V_I^{\min}$, and the net present value of transfers are increasing when the MMLR increases the option value further. Hence, the costs of delaying are strictly larger than an immediate, but minimal intervention.

5.4 Optimal Interventions

The policy we described in the last proposition is not necessarily optimal, because it might be better to save on transfers by delaying the intervention. In particular, the MMLR can choose to increase the size of intervention V_I and rely on an announcement effect to speed up the market recovery. Below, we will first study how the optimal intervention time depends on the importance of the market, captured by θ , and then discuss with some numerical examples the effects of other key parameters (e.g. π and λ) on the design of optimal intervention.

First, when a market is deemed important (small θ), an immediate intervention at a minimum scale is optimal.

Proposition 14. *Suppose $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\}$. There exists a cut-off point $\underline{\theta} > 1/2$ such that it is optimal to intervene immediately for all $\theta \in [1/2, \underline{\theta}]$.*

The proof is in the appendix. Quite interestingly, the proof depends heavily on the limits of the announcement effect, in particular on there being a fixed cost for delaying the

intervention, but still having continuous trade. On the other hand, when a market is not so important (large θ), it is optimal not to intervene and let the market freeze persist.

Proposition 15. *The optimal policy has $T \rightarrow \infty$ for $\theta \rightarrow 1$.*

The proof is in the appendix. Given these two propositions, one would expect that for intermediate values of θ , it is optimal to have a delayed intervention. When the intervention is delayed, equilibria have generically partial trading before the intervention. Hence, welfare cannot be computed analytically anymore, as our linear differential equations have time-dependent coefficients. While further characterizing the optimal intervention is analytically infeasible, we can numerically solve for the optimal policy as a function of key parameters. Table 1 again gives parameter values for our numerical example.

First, suppose there is a quality shock that drives π to $\pi_0 = 0.30$ and thus there is a permanent market freeze. The optimal intervention depends on the importance of the market captured by the parameter θ . Resurrecting the market is more important when θ is close to 0.5, and less important when close to 1. Figure 7 plots the optimal policy as a function of θ . The optimal intervention can be divided into three regions with respect to θ . When θ is small, an immediate intervention at a minimum scale is optimal. The market will function continuously. For intermediate values of θ , it is optimal to delay the intervention and provide a positive option value (i.e., $T > 0, V_I > 0$). Over this region, the optimal intervention time is increasing in θ , while the size of the intervention stays basically constant at an optimal, positive option value. Whenever the MMLR delays, it will rely on the announcement effect. In particular, $0 < \tau_1 < \tau_2 < T$ so that there is first a partial recovery, followed by a full market recovery before the intervention. Finally, for high values of θ , it is optimal to not to intervene at all (i.e. $T = +\infty, V_I = 0$).

We then study how the optimal intervention time T responds to the severity of the quality shock (captured by $\pi_{\min} - \pi(0)$), the relative importance of resurrecting the market (captured by θ), and the market frictions (captured by λ). Figure 8 uses iso-value curves to illustrate the optimal intervention time T for different combinations of π shocks and θ . For a given λ , the optimal T is increasing in both the π shock and θ . If the quality shock is small and a market is deemed important, it is optimal to intervene immediately to ensure that the market functions continuously. For sufficiently high quality shock and unimportant markets, it is optimal to tolerate a market freeze and make no intervention. Finally, for markets with high trading frictions (i.e., small λ), it is optimal to intervene earlier because MMLR cannot rely on announcement effects to resurrect markets.

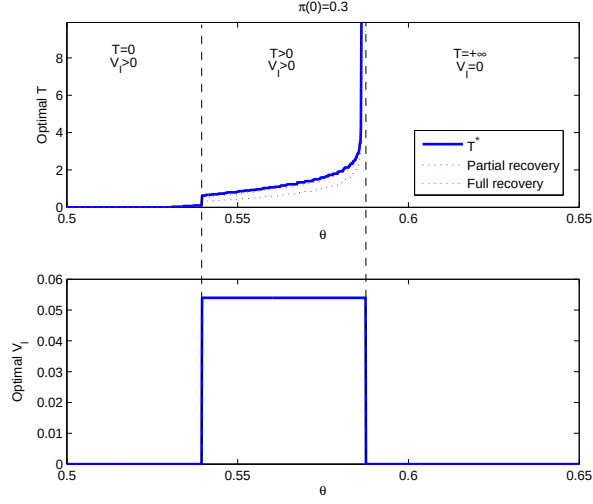


Figure 7: Optimal (T, V_I) as a function of θ ($\pi(0) = 0.30$)

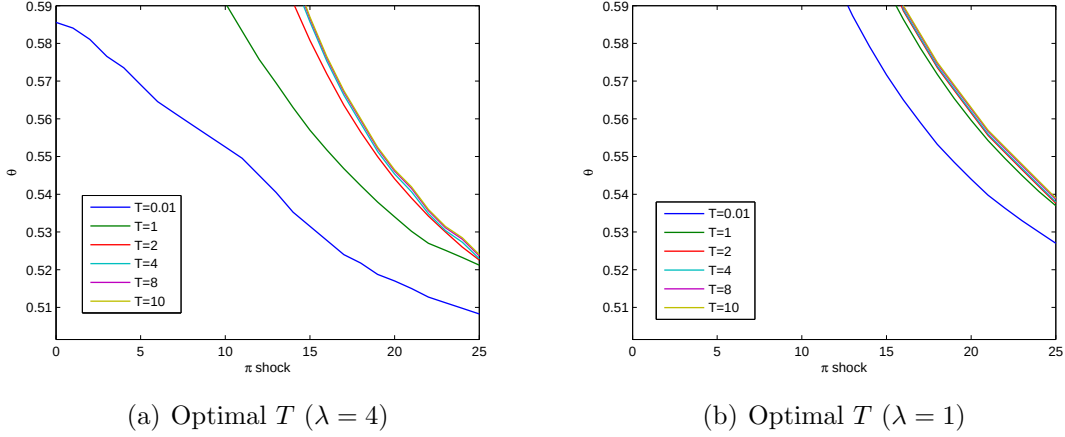


Figure 8: Optimal T as a function of $\pi(0)$, θ and λ

6 Conclusion

We have shown in this paper that a MMLR can resolve a market freeze, if he buys a sufficient amount of assets. Most interestingly, there can be an announcement effect: simply announcing the intervention can induce investors to start trading again. Depending on the size of the shock to the average asset quality, the announcement effect may require a larger than the minimal intervention. This opens up a trade-off between the size and the timing of the intervention.

Policy makers often discuss about having financial markets function continuously. We

have shown that a central bank or government facing such a goal should not rely on the announcement effect, but intervene immediately at a minimum scale. We have also shown here that indeed one can rationalize continuous markets as an optimal policy goal, if the cost of public funds for the intervention are sufficiently small. This stands in stark contrast to the literature that looks at marketmaking for liquidity reasons. There, immediate intervention after a liquidity shock is never optimal (see for example [18]).

A stark assumption in our analysis makes the shock to the quality of the asset permanent. With a random recovery time for $\pi(0)$ to jump back to the original level our results might change. If the initial shock is small and recovery relatively likely, the market might just function continuously on its own. Also, there might be an incentive now to delay the intervention – even if the MMLR is required to ensure continuous markets. Delaying saves costs in expected terms, even if this requires an increase in the size of the intervention V_I . Also, once the asset quality recovers, the MMLR has the option to sell back some of the assets it has acquired. This might induce it to intervene earlier as now the expected costs of transfers have decreased.

An interesting detail of the announcement effect is that there is a time-consistency problem. Suppose for the moment that investors believe the announcement that the intervention will take place at some time $T > 0$ and start trading. The MMLR has then an incentive at T to surprise the market by postponing the intervention for a small interval. Even if the market would freeze for a short period, this could mean saving costs by delaying the intervention. Investors will then view such announcements as not credible. To solve this time-consistency problem, the MMLR might have to spread out purchases over time, thereby reducing the gains from delaying the intervention.

We have not looked at another, related problem. The quality shock is exogenous in our model. Suppose, however, that investors can create new assets. Anticipating that a MMLR will resurrect the market, investors will have an incentive to create lemons. Hence, a moral hazard problem arises from intervening in the event of a market freeze. The MMLR could counteract these incentives by randomizing between intervening or not. We leave a detailed analysis of the last two issues for future work.

7 Appendix

Proof of Lemma 3:

Proof. Let T be the time of intervention by the MMLR and assume that the MMLR removes I assets.

Step 1: $\tilde{\pi}(t)$ increases when there is no trade ($\gamma = 0$).

For any no trade interval $[\tau, \tau']$ with $\tau \geq T$, flows at time $t \in [\tau, \tau']$ are given by

$$\begin{aligned}\dot{\mu}_\ell(t) &= 0 \\ \dot{\mu}_s(t) &= \kappa\mu_o(t) \\ \dot{\mu}_o(t) &= -\kappa\mu_o(t).\end{aligned}$$

Therefore, the measure of sellers is given by

$$\mu_s(t) = S\pi(0) (1 - e^{-\kappa(t-\tau)}) + \mu_s(\tau)e^{-\kappa(t-\tau)}$$

and the fraction of good assets on the market is given by

$$\tilde{\pi}(t) = \frac{S\pi(0) (1 - e^{-\kappa(t-\tau)}) + \mu_s(\tau)e^{-\kappa(t-\tau)}}{S\pi(0) (1 - e^{-\kappa(t-\tau)}) + \mu_s(\tau)e^{-\kappa(t-\tau)}(1 - \pi(0))S - I}.$$

We have that up to a constant

$$\frac{\partial \tilde{\pi}(t)}{\partial t} = -\kappa e^{-\kappa(t-\tau)}(1 - \pi(0))(S - I) (\mu_s(\tau) - S\pi(0)) > 0 \quad (7.1)$$

as $\mu_s(\tau) < S\pi(0)$.

Step 2: $\tilde{\pi}(t)$ declines when there is trade ($\gamma = 1$).

The law of motion is given by

$$\begin{aligned}\dot{\mu}_\ell &= 0 \\ \dot{\mu}_s(t) &= \kappa\mu_o(t) - \gamma\lambda\mu_s(t)\mu_b \\ \dot{\mu}_o(t) &= -\kappa\mu_o(t) + \gamma\lambda\mu_s(t)\mu_b.\end{aligned}$$

Therefore, the measure of sellers is given by

$$\mu_s(t) = S\pi(0) \frac{\kappa}{\kappa + \gamma\lambda\mu_b} (1 - e^{-(\kappa + \gamma\lambda\mu_b)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa + \gamma\lambda\mu_b)(t-\tau)}$$

Again, for an interval with trade $[\tau, \tau']$ with $\tau \geq T$ we have

$$\tilde{\pi}(t) = \frac{S\pi(0)\frac{\kappa}{\kappa+\gamma\lambda\mu_b}(1 - e^{-(\kappa+\gamma\lambda\mu_b)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa+\gamma\lambda\mu_b)(t-\tau)}}{S\pi(0)\frac{\kappa}{\kappa+\gamma\lambda\mu_b}(1 - e^{-(\kappa+\gamma\lambda\mu_b)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa+\gamma\lambda\mu_b)(t-\tau)} + (1 - \pi(0))S - I}$$

which gives up to constant

$$\frac{\partial \tilde{\pi}(t)}{\partial t} = -(\kappa + \gamma\lambda\mu_b)e^{-(\kappa+\gamma\lambda\mu_b)(t-\tau)} [(1 - \pi(0))S - I] \left(\mu_s(\tau) - S\pi(0)\frac{\kappa}{\kappa + \gamma\lambda\mu_b} \right). \quad (7.2)$$

If there has been continuous trade from $t = 0$ until τ , i.e. $\gamma(t) = 1$ for all $t \in [0, \tau]$, we have that $\mu_s(\tau) = \mu_s(0) = S\pi(0)\frac{\kappa}{\kappa+\lambda\mu_b}$. Hence, $\tilde{\pi}(t)$ is constant as long as $\gamma = 1$.

If there has not been trade at some time before τ , i.e. $\gamma(t) = 0$ for some $[t_1, t_2] \subset [0, \tau]$, it must be the case that $\mu_s(\tau) > \mu_s(0)$.

Hence, $\tilde{\pi}(t)$ declines independent of trading behavior before the intervention, since $\mu_s(\tau) \geq \mu_s(0) = S\pi(0)\frac{\kappa}{\kappa+\lambda\mu_b}$. $\square$

Proof of Proposition 14:

Proof. Denote by $V(T = 0)$ and $c(T = 0)$ the benefits and costs of continuous trade. Note that we know already that the optimal policy conditional on continuous trade is given by $T = 0$ and a minimum intervention.

Step 1: For θ sufficiently close to $1/2$, $T = \infty$ cannot be optimal.

If $T = \infty$ were optimal, we have that

$$V(T = \infty) > V(T = 0) - \frac{2\theta - 1}{1 - \theta}c(T = 0). \quad (7.3)$$

Since $V(T = \infty) < V(T = 0)$, for θ sufficiently close to $1/2$ this inequality is violated.

Step 2: Denote the optimal policy by $\Psi(\theta)$ for all $\theta \in (1/2, 1)$. Then, $V(\Psi(\theta)) \rightarrow V(T = 0)$ for $\theta \rightarrow 1/2$.

Since $\Psi(\theta)$ is the optimal policy, we have that

$$V(T = 0) - \frac{2\theta - 1}{1 - \theta}c(T = 0) < V(\Psi(\theta)) - \frac{2\theta - 1}{1 - \theta}c(\Psi(\theta)) \quad (7.4)$$

where it must be the case that $c(\Psi(\theta)) < c(T = 0)$. Hence,

$$0 < V(T = 0) - V(\Psi(\theta)) < \frac{2\theta - 1}{1 - \theta} [c(T = 0) - c(\Psi(\theta))], \quad (7.5)$$

and the result follows.

Step 3: The optimal policy $\Psi(\theta)$ satisfies $T(\theta) \rightarrow 0$ and $V_I(\Psi(\theta)) \rightarrow 0$ for $\theta \rightarrow 1/2$.

Suppose to the contrary that $\liminf T(\theta) = \underline{T} > 0$ for $\theta \rightarrow 1/2$. Then, it must be the case that $V(\Psi(\theta_n)) \rightarrow V(T = 0)$. This implies that $\tau_2(T) \rightarrow 0$. As $T(\theta_n) > \underline{T}$, it must be the case that

$$\liminf c(\Psi(\theta_n)) > c(T = 0) + c(V_I^{\min}) > c(T = 0). \quad (7.6)$$

But this implies that for θ sufficiently close to $1/2$, $c(T = 0)$ is cheaper. Hence, $\Psi(\theta)$ cannot be the optimal policy, a contradiction.

The second part of the statement follows from an analogous argument.

Step 4: For any sequence $\theta_n \rightarrow 1/2$, there exists some N such that $T(\theta_n|\Psi) = 0$.

We have that for the optimal policy $\Psi(\theta_n)$, $T \rightarrow 0$ and $V_I \rightarrow 0$. We show next that this implies that for θ_n close enough to $1/2$, there is no trade in the interval $[0, T(\theta_n)]$.

Suppose that $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\}$ and let $T > 0$. For a minimum intervention, i.e. $V_I = 0$, we have that

$$\Gamma(0) = \tilde{\pi}(0)(v_o - v_s) + (1 - \tilde{\pi}(0))(v_\ell(0) - v_s) < 0. \quad (7.7)$$

The value of a lemon for all $t < T$ can at most be $v_\ell(t) < \frac{\lambda}{\lambda+r}v_s + V_I$. For $V_I < V_I^{\min}$, we also have

$$\Gamma(t) = \tilde{\pi}(t)(v_o - v_s) + (1 - \tilde{\pi}(t))(v_\ell(t) - v_s) < 0, \quad (7.8)$$

for all t in some interval $[0, \mathcal{T})$ where $\mathcal{T} > 0$.

Given $\tilde{\pi}(0)$, we can then choose N large enough such that T and V_I are sufficiently close to 0 to satisfy

$$\Gamma(T^-) < \tilde{\pi}(T^-)(v_o - v_s) + (1 - \tilde{\pi}(T^-)) \left(\frac{r}{\lambda+r}v_s + V_I \right) < 0. \quad (7.9)$$

Then, $\Gamma(t) < 0$ for all $[0, T)$. Hence, there is no trade before the intervention for any candidate policy that is better than $T = 0$ and where θ is sufficiently close to $1/2$.

Step 5: We can then use an earlier result that has shown that conditional on there being

no trade before the intervention, it is optimal to intervene immediately for θ close enough to $1/2$. This completes the proof. $\square$

Proof of Proposition 15:

Proof. Suppose not. Then $\limsup_{\theta \rightarrow 1} T = \bar{T} < \infty$. This implies that

$$c(\Psi) \geq p_{\min} I_{\min} e^{-r\bar{T}} > 0. \quad (7.10)$$

We have that the benefits of any policy are bounded by $V(T = 0)$. Hence, for any optimal policy we have that

$$V(T = 0) - V(T = \infty) \geq V(\Psi(\theta)) - V(T = \infty) \geq \frac{2\theta - 1}{1 - \theta} c(\Psi(\theta)) \geq \frac{2\theta - 1}{1 - \theta} p_{\min} I_{\min} e^{-r\bar{T}}. \quad (7.11)$$

Letting $\theta \rightarrow 1$ violates this inequality. $\square$

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